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GEOMETRICAL ASPECT OF TOPOLOGICALLY TWISTED 2-DIMENSIONAL CONFORMAL SUPERALGEBRA

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ABSTRACT

We study the topologically twisted osp(2|2)⊕osp(2|2) conformal superalgebra. The algebra includes the Lagrangians which are intrinsic to the topological field theory and composed of fermionic generators. Studying the Lagrangians through a gauge system of osp(2|2)⊕osp(2|2) , geometrical features inherent to the algebra are revealed: a moduli space associated with the algebra is derived and the crucial roles which the fermionic generators play in the moduli space are clarified. It is argued that there exists a specific relation between the topological twist and the moduli problem through a geometrical aspect of the algebra.

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1 Introduction

In the recent progress of the quantum field theory (QFT), the detection of the cohomological field theory may be most fascinating development. The theory is a kind of the topological QFT which deal with topological invariants, and has been pioneered by E. Witten[1]. We refer to the cohomological field theory as TFT in the present paper and will focus on it. The theory has some characteristic properties on the construction and has distinct framework. Therefore, many energetic researches in TFT have been done[2] and then TFT has been proved to be a real solid methodology in QFT. A few substantial problems associated with TFT still remain to be solved, for example, about the topological twist, however. In the conformal field theory (CFT), the toplogical twist of N=2 CFT is performed through a redefinition of the energy-momentum tensor of N=2 theory[3], which generates the bosonic CFT models of vanishing central charge with hidden fermionic (topological) symmetry. In relation to the topological twisting mechanism, the different twistings of the same model yield the different moduli problems, respectively, which are related through the mirror symmetry[4] as the explicit example of twisting in general. For another example, the topological gauged WZW models[5] are composed of two different gauge fixing procedures from the same bosonic model, not necessarily twisting of the N=2 supersymmetry[6] of Kazama-Suzuki model[7], and in this case there surely exists the mirror symmetry.

There are two typical stand points for constructing TFT, i.e. topological twisting and BRST gauge fixing. Both approaches result in the so-called moduli problem[8][9]. In either case, the remarkable characteristic is that the Lagrangian is described as $\mathcal{L} = \{\mathcal{Q},\ \star\}$, where $\mathcal{Q}$ is the fermionic operator of nilpotency, i.e. the so-called topological symmetry. In terms of the ordinary QFT words, $\mathcal{L}$ is just composed of the BRST gauge fixing and the FP ghost terms, and the $\mathcal{Q}$ corresponds to the BRST operator. Because of the BRST-exact form of $\mathcal{L}$, every correlation function is independent of the coupling factor as a consequence of which the leading contribution to the path-integral is only the classical configuration of the fields, i.e. zero mode. This zero mode configuration is

associated with some moduli space. In the BRST approach, the relation between the moduli problem and TFT may be comparatively clear owing to the intrinsic constructing procedure where some moduli problem can be settled as the gauge fixing condition. In the topological twisting formalism, the above relation is not much clear, on the contrary. It seems that there has not been a common recognition on what the topological twist is really doing.

In the present paper, we concentrate on N=2 finite-dimensional superalgebra in two dimensions, perform the topological twist on such a superalgebra, and show a characteristic property associated with the topological twist by discussing the twisted superalgebra, i.e. the so-called topological algebra, through a gauge system. First, a geometrical feature inherent to the algebra is revealed, and then it is argued that there exists a specific relation between the topological twist and the moduli problem through a geometrical aspect of the algebra.

In the next section, we decide on osp(2|2)⊕osp(2|2) as N=2 finite-dimensional superalgebra and perform the topological twist on osp(2|2)⊕osp(2|2) , so that topological algebra is obtained. In section 3, three types of the TFT's Lagrangian are found in the topological algebra. One of the three types of Lagrangian is focused on and the field configuration is investigated in the case of zero-limit of the coupling factor on the path-integral by considering a gauge system. It is shown that this configuration is indeed a moduli space of flat connections associated with the topological algebra, and this fact originates from vanishing Noether current. In section 4, a geometrical aspect of the fermionic charges is discussed. Under the weak coupling limit, the total Lagrangian which is a linear combination of the three Lagrangians is regarded as the Laplacian operator on the moduli space, and the fermionic charges as Fredholm operators. Taking account of these facts, the moduli space which is obtained formally in sect.3 is made more visual. It is also shown that the index of these operators could be derived if proper support in the moduli space can be defined. Lastly in sect.4, we discuss the triviality of the path-integral and obtain a non-trivial TFT's observable. The fact will supports the argument developed in the present paper. In the final section, It is claimed that

the algebra has a specific relation with the moduli problem and the some remark about the vanishing Noether current which plays a crucial role in the following discussions is mentioned.

2 Procedure of Topological Twist

2-1 osp(2|2)⊕osp(2|2) Algebra

The first issue is the specification of N=2 finite-dimensional superalgebra in 2-dimensions on which the topological twist will be performed. The topological twist usually means the mixing of the representation space of the internal symmetry group of the supersymmetry with that of the symmetry group with respect to the space-time, i.e. spinor space. On manifolds, the latter symmetry is local. Consequently, the former symmetry must also be local. The situation is allowed in the case of the conformal supersymmetry alone. On the contrary, the other types of the super-extended algebra, that is, super-Poincaré or super-(anti-)de-Sitter, must not be adapted for the present case because it is not possible to deal with its internal symmetry as a purely geometrical object in contrast to the conformal case. In the first place, the topologically twisted super-Poincaré algebra is incomplete from the geometrical view point [11][12].

What we are next interested in is the finite-dimensional conformal superalgebras. The finite-dimensional simple Lie superalgebras are fully investigated[13] and all the finite-dimensional conformal superalgebras in two dimensions are shown[14]. The 4-types of all the algebras must be eliminated from the physical view point;s_2, osp(2,1|N), su(1,1|1,1) and d(1,2;α)[14]. The 4-types are unsuitable also for the present case, either because there exists no anti-commutator of the supercharges or because the supersymmetry is in the representations of integer spin. We will soon understand the reason why in the forthcoming contexts. After all, the possible finite-dimensional conformal superalgebras in 2-dimentions are then osp(N|2) (N≥0), su(N|1,1) (N≥2), f_4 and g_3. Moreover, it is the N=2 case that we are interested in. In the case, the remaining card is osp(2|2)

alone. Therefore, osp(2|2) is the unique solution for performing the topological twist on finite-dimensional N=2 superalgebra in two dimensions.

The internal symmetry group of Osp(2|2)⊗Osp(2|2) in relation to $(2,2)$ supersymmetry is SO(2)⊗SO(2), and Osp(2|2) is required to be compact so that its Cartan-Killing form is positive definite, while the super Lie group Osp(2|2) is generally not compact. osp(2|2)⊕osp(2|2) conformal superalgebra on which the twisting operation will be made to form a corresponding topological algebra is then confined to the two dimensional Lorentzian manifold with the local Lorentz metric in the light-cone coordinates: $g^{z\bar{z}} = g^{\bar{z}z} = -2$, $g_{z\bar{z}} = g_{\bar{z}z} = -1/2$ and $g^{zz} = g^{\bar{z}\bar{z}} = g_{zz} = g_{\bar{z}\bar{z}} = 0$. This (2,2) superalgebra contains two types of complex Weyl spinorial charges Q, $\bar{Q}$; S, $\bar{S}$, where "$-$" means the Dirac conjugation $\bar{Q} = Q^\dagger\gamma^0$ in which $\gamma^0 = i\sigma^2$ and incidentally $\gamma^1 = \sigma^1$, $\gamma^5 = -\gamma^0\gamma^1 = \sigma^3$, or equivalently $\gamma^z = \gamma^0 + \gamma^1$, $\gamma^{\bar{z}} = \gamma^0 - \gamma^1$ in the light-cone coordinates. These supercharges are two component spinors, for example $Q = (Q_+,\ Q_-)^t$, where "$+,-$" mean spinor indices describing "left" and "right" moving , respectively, with respect to the local Lorentz coordinates $(z, \bar{z})$. These indices are raised and lowered by a metric in spinor space given by the charge conjugation matrix $C = \gamma^0$: $\eta^{+-} = \eta_{-+} = -1$, $\eta^{-+} = \eta_{+-} = 1$ and $\eta^{++} = \eta^{--} = \eta_{++} = \eta_{--} = 0$.

We can leave out the conjugate parts of the bra-ket bosonic relations with respect to the complex supercharges of osp(2|2)⊕osp(2|2) as follows:

$$
\begin{aligned}
&[S, P_a] = \gamma_a Q, \qquad [S, D] = -\frac{1}{2}S, \qquad [S, M] = -\frac{1}{2}\gamma_5 S,\\
&[Q, K_a] = -\gamma_a S, \qquad [Q, D] = \frac{1}{2}Q, \qquad [Q, M] = -\frac{1}{2}\gamma_5 Q,\\
&[S, A] = -i\frac{1}{4}\gamma_5 S, \qquad [S, V] = -i\frac{1}{4}S,\\
&[Q, A] = i\frac{1}{4}\gamma_5 Q, \qquad [Q, V] = -i\frac{1}{4}Q.
\end{aligned} \tag{2.1}
$$

If we want to get these conjugate parts of eqs.(2.1), we must pay attention to the fact that the representation of the body so(2)⊕sp(2) of osp(2|2) are anti-Hermitian where the anti-Hermitian character of the representation of sp(2) actually leads to the positivity of the Cartan-Killing form of osp(2|2).

Ordinary (2,2) supersymmetry which is free from the central charges is direct sum of (2,0) and (0,2) and the corresponding part in osp(2|2)⊕osp(2|2) reads

$$
\{Q_+, \bar{Q}_+\} = iP_z, \qquad \{S_+, \bar{S}_+\} = -iK_z,
$$
$$
\{Q_-, \bar{Q}_-\} = iP_{\bar{z}}, \qquad \{S_-, \bar{S}_-\} = -iK_{\bar{z}}. \tag{2.2}
$$

While the super-extended conformal algebra has no central charge, there are mixing parts, instead, in the relations between the supercharges, and consequently the decomposition mentioned above does not exist. The mixing part of (2,0) and (0,2) in osp(2|2)⊕osp(2|2) is

$$
\begin{aligned}
\{Q_+, \bar{S}_-\} &= i(M - D) + 2(A - V),\\
\{Q_-, \bar{S}_+\} &= i(M + D) + 2(A + V),\\
\{\bar{Q}_+, S_-\} &= i(M - D) - 2(A - V),\\
\{\bar{Q}_-, S_+\} &= i(M + D) - 2(A + V).
\end{aligned} \tag{2.3}
$$

The property will play an important role in the forthcoming contexts.

The bosonic generators of osp(2|2)⊕osp(2|2) are as follows: P_a, K_a, M, D, A and V are translation, conformal-translation, Lorentz, Weyl, chiral so(2) and internal so(2), respectively. The finite two-dimensional conformal algebra composed of these bosonic generators alone is

$$
[P_a, M] = \epsilon_{ab} P^b, \qquad [P_a, D] = P_a, \qquad [K_a, D] = -K_a,
$$
$$
[K_a, M] = \epsilon_{ab} K^b, \qquad [K_a, P_b] = 2(\epsilon_{ab} M - \delta_{ab} D), \tag{2.4}
$$

where ϵ are $\epsilon^{z\bar{z}} = -\epsilon^{\bar{z}z} = -2$, $\epsilon_{z\bar{z}} = -\epsilon_{\bar{z}z} = 1/2$ and $\epsilon^{zz} = \epsilon^{\bar{z}\bar{z}} = \epsilon_{zz} = \epsilon_{\bar{z}\bar{z}} = 0$.

Lastly in the presentation of osp(2|2)⊕osp(2|2) algebra, let us comment on the naming of the generators of osp(2|2)⊕osp(2|2) . In sect.3 where a pure gauge theory of osp(2|2)⊕osp(2|2) on two dimensional manifold will be considered, the naming, for instance, P as translation, is perfectly formal. If not, the general coordinate transformations must exist in the system and then the theory may become empty as well as the

ordinary 2D conformal supergravity theories[10].

2-2 Topological Twist

We are now in a position to perform topological twisting of the algebra. Topological twist is usually a kind of mixing which results in identification of the representation space of internal symmetry group of N=2 supersymmetry with that of the local Lorentz group. It is easy to perform twisting of the algebra to get the topological algebra. Most of all we have to do is to replace Q, $\bar{Q}$, S, and $\bar{S}$ with Q^+, Q^-, S^+, and S^-, respectively. The indices "$+,-$" are raised and lowered with the same metric as for the indices $\alpha;\beta$ of $C_{\alpha\beta}$ and Q_α. That is, the complex Weyl spinors φ_α, $\bar{\varphi}_\alpha$ are substituted for ${\varphi_\alpha}^+$, ${\varphi_\alpha}^-$:

$$\varphi_\alpha = \frac{i}{\sqrt{2}}{\varphi_\alpha}^+, \quad \bar{\varphi}_\alpha = \frac{i}{\sqrt{2}}{\varphi_\alpha}^-. \tag{2.5}$$

The remaining manipulations are as follows. The fermionic charges $Q^+ = ({Q_+}^+, {Q_-}^+)^t$ have become ((0,0)-form , (0,1)-form), and $Q^- = ({Q_+}^-, {Q_-}^-)^t$ with ((1,0)-form , (0,0)-form), *idem* $S^\pm$. Then we have to modify the definitions of local Lorentz M and Weyl D generators so that the four (0,0)-form fermionic generators of supersymmetry have no charge with respect to these two bosonic generators. We have put the representation space accompanied with the internal symmetry group SO(2)⊗SO(2) upon the space of spinor. The modified M, D generators must be direct sums with so(2)⊕so(2) generators V and A, respectively. The solution to this constraint resolves into

$$\tilde{M} = M + 2iV, \quad \tilde{D} = D + 2iA. \tag{2.6}$$

These modified generators then satisfy the following relations:

$$[{\triangle_\pm}^\pm, \tilde{M}] = 0, \quad [{\triangle_\pm}^\pm, \tilde{D}] = 0, \tag{2.7}$$

where $\triangle$ means both Q and S.

There appear some problems about the closure of the modified algebra, however. The generators A and V have been put upon D and M, respectively and the modified

algebra which contains $\tilde{M}$ and $\tilde{D}$ must not contain A and V. In fact, the modified algebra contains subtle relations:

$$\{Q_-{}^+, S_+{}^-\} = i(\tilde{M} + \tilde{D}) - 4i(A + V),$$
$$\{Q_+{}^-, S_-{}^+\} = i(\tilde{M} - \tilde{D}) + 4i(A - V). \qquad (2.8)$$

We can avoid the above relations (2.8) as in the followings. Here it is necessary to omit another generators with regard to eqs.(2.8), if this modified algebra still obeys the closure property for the generators of the gauge symmetry. In this point of view, the four fermionic generators $Q_+{}^-$, $Q_-{}^+$, $S_+{}^-$ and $S_-{}^+$ do not induce the gauge transformations generated by both $i(\tilde{M} + \tilde{D}) - 4i(A + V)$ and $i(\tilde{M} - \tilde{D}) + 4i(A - V)$. There are two alternatives, that is, the case in which the left chiral charges $Q_+{}^-, S_+{}^-, P_z$, and K_z vanish, or the case in which the right chiral charges $Q_-{}^+, S_-{}^+, P_{\bar{z}}$, and $K_{\bar{z}}$ vanish, without any compensation procedure, that is, all gauge fields and parameters of these four generators are assured to vanish. The second case is adapted here. In sect.4, it will be shown that a moduli space derived from either case is reduced to that associated with an intersection part of both cases.

The twisting procedure is explained in terms of the gauge fields of the corresponding symmetry osp(2|2)⊕osp(2|2) . Let us introduce the gauge field $\mathbf{a}$ which is Lie superalgebra-valued 1-form of osp(2|2)⊕osp(2|2) in the form:

$$\begin{aligned} \mathbf{a}_\mu = e^a_\mu P_a \quad &+ \quad f^a_\mu K_a + \omega_\mu M + b_\mu D \\ &+ \quad a_\mu A + v_\mu V + \bar{\psi}_\mu Q + \bar{Q}\psi_\mu + \bar{\phi}_\mu S + \bar{S}\phi_\mu, \end{aligned} \qquad (2.9)$$

as well as transformation parameter τ defined by

$$\begin{aligned} \tau = \xi^a_P P_a \quad &+ \quad \xi^a_K K_a + \lambda_l M + \lambda_d D \\ &+ \quad \theta_a A + \theta_v V + \bar{\varepsilon} Q + \bar{Q}\varepsilon + \bar{\kappa} S + \bar{S}\kappa. \end{aligned} \qquad (2.10)$$

Using the gauge fields and parameters, the above mentioned topological twist and additional manipulations can be described as follows: eqs.(2.6) mean

$$v_\mu = 2i\omega_\mu, \quad a_\mu = 2ib_\mu, \qquad (2.11)$$

and elimination of the generators $Q_-{}^+$, $S_-{}^+$, $P_{\bar{z}}$ and $K_{\bar{z}}$ means

$$\begin{aligned} \phi_{\mu+}{}^- = 0 = \psi_{\mu+}{}^-, &\qquad \kappa_+{}^- = 0 = \varepsilon_+{}^-, \\ e_\mu^{\bar{z}} = 0 = f_\mu^{\bar{z}}, &\qquad \xi_P^{\bar{z}} = 0 = \xi_K^{\bar{z}}. \end{aligned} \tag{2.12}$$

Under the conditions we are led to

$$\delta\phi_{\mu+}{}^+ \sim \delta\phi_{\mu-}{}^-, \quad \delta\psi_{\mu+}{}^+ \sim \delta\psi_{\mu-}{}^-. \tag{2.13}$$

Accordingly, we have the following identifications:

$$\begin{aligned} \psi_{\mu+}{}^+ = -\psi_{\mu-}{}^- \equiv -\psi_\mu, &\quad \phi_{\mu+}{}^+ = -\phi_{\mu-}{}^- \equiv -\phi_\mu, \\ \varepsilon_+{}^+ = -\varepsilon_-{}^- \equiv -\varepsilon, &\quad \kappa_+{}^+ = -\kappa_-{}^- \equiv -\kappa, \end{aligned} \tag{2.14}$$

which read without loss of generality

$$Q \equiv Q_+{}^+ + Q_-{}^-, \quad S \equiv S_+{}^+ + S_-{}^-. \tag{2.15}$$

Taking into account all these additional conditions with respect to the topological twist on the original osp(2|2)$\oplus$osp(2|2) , we get the gauge connection $\mathbf{a}$:

$$\begin{aligned} \mathbf{a}_\mu = e_\mu^z P_z + f_\mu^z K_z &+ \omega_\mu \tilde{M} + b_\mu \tilde{D} \\ &- \frac{1}{2}(Q_+{}^- \psi_{\mu-}{}^+ + S_+{}^- \phi_{\mu-}{}^+ + \psi_\mu Q + \phi_\mu S), \end{aligned} \tag{2.16}$$

and transformation parameter τ:

$$\begin{aligned} \tau = \xi_P^z P_z + \xi_K^z K_z &+ \lambda_l \tilde{M} + \lambda_d \tilde{D} \\ &- \frac{1}{2}(Q_+{}^- \varepsilon_-{}^+ + S_+{}^- \kappa_-{}^+ + \varepsilon Q + \kappa S), \end{aligned} \tag{2.17}$$

respectively.

After all, the generators in eqs.(2.16) (2.17) obey the following relations:

$$[S, P_z] = Q_+{}^-, \quad [Q_+{}^-, \tilde{D}] = Q_+{}^-, \quad [Q_+{}^-, \tilde{M}] = -Q_+{}^-,$$

$$[Q, K_z] = -S_+{}^-, \quad [S_+{}^-, \tilde{D}] = -S_+{}^-, \quad [S_+{}^-, \tilde{M}] = -S_+{}^-,$$

$$\{Q, Q_+{}^-\} = -2iP_z, \quad \{S, S_+{}^-\} = 2iK_z, \quad \{Q, S\} = -4i\tilde{M},$$
$$[P_z, \tilde{M}] = -P_z, \quad [P_z, \tilde{D}] = P_z,$$
$$[K_z, \tilde{M}] = -K_z, \quad [K_z, \tilde{D}] = -K_z, \tag{2.18}$$

and the gauge connections (2.16) satisfy the following transformation rules:

$$\delta\psi_\mu = \partial_\mu \varepsilon,$$
$$\delta\phi_\mu = \partial_\mu \kappa,$$
$$\delta\psi_{\mu-}{}^+ = \mathcal{D}_\mu \varepsilon_-{}^+ + \xi_P^z \phi_\mu + (\lambda_l - \lambda_d)\psi_{\mu-}{}^+ - e_\mu^z \kappa,$$
$$\delta\phi_{\mu-}{}^+ = \mathcal{D}_\mu \kappa_-{}^+ - \xi_K^z \psi_\mu + (\lambda_l + \lambda_d)\phi_{\mu-}{}^+ + f_\mu^z \varepsilon,$$
$$\delta e_\mu^z = \mathcal{D}_\mu \xi_P^z + (\lambda_l - \lambda_d) e_\mu^z - \frac{i}{4}(\varepsilon\psi_{\mu-}{}^+ - \psi_\mu \varepsilon_-{}^+),$$
$$\delta f_\mu^z = \mathcal{D}_\mu \xi_K^z + (\lambda_l + \lambda_d) f_\mu^z + \frac{i}{4}(\kappa\phi_{\mu-}{}^+ - \phi_\mu \kappa_-{}^+),$$
$$\delta\omega_\mu = \partial_\mu \lambda_l + \frac{i}{4}(\kappa\psi_\mu - \varepsilon\phi_\mu),$$
$$\delta b_\mu = \partial_\mu \lambda_d, \tag{2.19}$$

where

$$\mathcal{D}_\mu \varepsilon_-{}^+ = (\partial_\mu - \omega_\mu + b_\mu)\varepsilon_-{}^+, \quad \mathcal{D}_\mu \kappa_-{}^+ = (\partial_\mu - \omega_\mu - b_\mu)\kappa_-{}^+,$$
$$\mathcal{D}_\mu \xi_P^z = (\partial_\mu - \omega_\mu + b_\mu)\xi_P^z, \quad \mathcal{D}_\mu \xi_K^z = (\partial_\mu - \omega_\mu - b_\mu)\xi_K^z, \tag{2.20}$$

The field strengths in relation to the discarded right chiral charges $Q_-{}^+$, $S_-{}^+$, $P_{\bar{z}}$ and $K_{\bar{z}}$ all vanish as expected. The resultant algebra (2.18) can be referred to as the topological algebra[11].

SO(2)⊗SO(2) symmetry still remains as global internal symmetry whose charge is the so-called ghost-number, the generators of which are defined by $G \equiv 2i(A - V)$, $\tilde{G} \equiv 2i(A + V)$. Here G and $\tilde{G}$ satisfy the following relations:

$$[G, Q_+{}^+] = Q_+{}^+, \quad [G, S_+{}^+] = -S_+{}^+,$$
$$[G, Q_+{}^-] = -Q_+{}^-, \quad [\tilde{G}, S_+{}^-] = S_+{}^-,$$

$$[\tilde{G}, Q_-{}^-] = Q_-{}^-, \quad [\tilde{G}, S_-{}^-] = -S_-{}^-, \tag{2.21}$$

where the other combinations are trivial. As a consequence of eqs.(2.21), indeed, it is natural to regard these generators G, $\tilde{G}$ as the ghost number operators. $Q_\pm{}^\pm$ and $S_+{}^-$ increase the ghost number by one unit, while $Q_+{}^-$ and $S_\pm{}^\pm$ decrease it by the same quantity. The assignment is consistent with the relations (2.18) (2.19).

In preparation for the forthcoming contexts, next, the description of the coordinate indices in the relations (2.18) must be simplified. First of all, the local Lorentz coordinates are substituted for spinor indices of the supercharges as follows:

$$Q_+{}^- = 2Q_z, \qquad S_+{}^- = 2S_z, \tag{2.22}$$

from the following relations of fermionic field φ:

$$\varphi_\alpha{}^\beta \;=\; \varphi_a(\gamma^a)_\alpha{}^\beta \;=\; \begin{array}{c c} & \begin{array}{cc} + & - \end{array} \\ \begin{array}{c} + \\ - \end{array} & \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \end{array} \varphi_z \;+\; \begin{array}{c c} & \begin{array}{cc} + & - \end{array} \\ \begin{array}{c} + \\ - \end{array} & \begin{pmatrix} 0 & 0 \\ -2 & 0 \end{pmatrix} \end{array} \varphi_{\bar{z}}, \tag{2.23}$$

where $\alpha(\beta)$ is "$+,-$" and "a" means "z, $\bar{z}$". The above supercharges are further substitutable as follows:

$$Q_z = q_z Q_c, \quad S_z = s_z S_c, \tag{2.24}$$

where q_z and s_z, carring the chiral idex; z, commute all generators in the algebra (2.18) and "c" means the "chiral". q_z and s_z are left chiral components of real vectors $q = (q_z, q_{\bar{z}})^t$, $s = (s_z, s_{\bar{z}})^t$, respectively. Therefore, Q_c and S_c are real generators. We then obtain a different description of the topological algebra (2.18):

$$[S, q^z P_z] = 2qQ_c, \quad [Q_c, \tilde{D}] = Q_c, \quad [Q_c, \tilde{M}] = -Q_c,$$

$$[Q, s^z K_z] = -2sS_c, \quad [S_c, \tilde{D}] = -S_c, \quad [S_c, \tilde{M}] = -S_c,$$

$$\{Q, Q_c\} = -\frac{i}{q} q^z P_z, \quad \{S, S_c\} = \frac{i}{s} s^z K_z, \quad \{Q, S\} = -4i\tilde{M},$$

$$[P_z, \tilde{M}] = -P_z, \quad [P_z, \tilde{D}] = P_z,$$

$$[K_z, \tilde{M}] = -K_z, \quad [K_z, \tilde{D}] = -K_z, \tag{2.25}$$

where $q = q^z q_z$ and $s = s^z s_z$.

We must note that the four generators $q^z P_z$, $s^z K_z$, Q_c and S_c still behave as holomorphic one forms because the real vectors q, s commute all generators in the algebra (2.25) and then the commutation relation with $\tilde{M}$ is still retained. It is a matter of course that, if 2-manifold M^2 is Hermitian with no boundary, four genarators $q^z P_z$, $s^z K_z$, Q_c and S_c could behave as zero forms, that is, they commute $\tilde{M}$, regarding $q^z (s^z)$ as the ordinary adjoint Dolbeault operator; $\partial^\dagger (= - * \bar{\partial} *)$ which satisfies the relation $[\tilde{M},\, \partial^\dagger] = -\partial^\dagger$. We must note that the scale dimensions can not be wiped out through $\partial^\dagger$, however.

3 Vanishing Noether Current

3-1 Brief Sketch of TFT

In the previous section, the twisted osp(2|2)⊕osp(2|2) algebra (2.25) has been obtained. We will show that the algebra (2.25) has the TFT's Lagrangians and the moduli space associated with the algebra can be derived. Our principal concern is now reduced to building up the TFT Lagrangian and quantizing it. Let us give a brief sketch of TFT[2] in preperation for the following discussions. The theory has some fermionic operator $\mathcal{Q}$ of nilpotency and the Lagrangian $\mathcal{L}$ is $\mathcal{Q}$-exact: $\mathcal{L} = \{\mathcal{Q}, \star\}$. The $\mathcal{Q}$-exact form of $\mathcal{L}$ shows that the energy-stress tensor of the Lagrangian is also $\mathcal{Q}$-exact. This is non-trivial because of no informaton whether $\delta/\delta g_{\mu\nu}$ and $\mathcal{Q}$ are commutative or not. All known examples are in the case, or rather, due to the $\mathcal{Q}$-exact form of the stress-energy tensor, it is possible to discuss the topological invariat correlation functions.

The TFT Lagrangian is allowed to have the gauge symmetries, in which the cohomological nature of TFT turns out to the equivariant cohomology: $\mathcal{Q}^2 = \tau_\phi$, where τ_ϕ means the gauge transformation with the parameter ϕ. The freedom of the gauge symmetry can be fixed by using BRST method as usual before the quantization of this

system. In TFT this freedom may be automatically fixed after the quantization through the path-integral, i.e. the reduction to some moduli space.

The correlation functions are independent of a coupling factor, because $\mathcal{L}$ is $\mathcal{Q}$-exact The zero limit of the coupling factor then induces the leading contribution of the path-integral with $\mathcal{L} = 0$. This configuration of the fields is associated with a moduli space. As a consequence, TFT makes a local field theory in the sense of a finite integration on the moduli space after quantization.

It is necessary to mention that the behavior of the path-integral measure is crucial in the case of full construction of TFT[6], e.g. in studies of the mirror symmetry[15][16]. The fact that, due to the coupling factor independence of correlation functions, the configuration of the theory could be reduced to the classical one, i.e. zero mode configuration, does not necessarily lead us to the conclusion that the theory could be regarded perfectly as classical in itself. The non-zero mode contribution must not be disregarded in the case of full construction of TFT. Anyhow, if we want to know the classical configuration alone, it is not necessary to take into account the non-zero mode as a quantum effect.

Due to the zero-mode contribution associated with the detailes of the moduli space, there is a close relation between the behavior of the measure and the triviality of the path-integral. The triviality depends on the characterestic of the TFT obsrevables which satisfy at least the following conditions: the gauge invariance, the metric idependence and the elements of $\mathcal{Q}$-cohomology group (rigorously speaking, the gauge invariance and the metric independence, modulo $\mathcal{Q}$-exact). In the path-integration under the limit, the integral measure must be reduced to that on the classical configuration and the fermionic number anomaly which corresponds to the local dimension of the classical configuration comes manifestly to the measure. Therefore, the non-trivial correlation functions must be given by the observables with the same fermionic number as the measure's. Consequently, we must need some proper obsevables, in other words, an assurance that the correlation functions are non-trivial. At this stage, it may not be possible to present such an assurance. In the next section, it will be shown that the path-integral in the

present case is non-trivial through discussion on a moduli space.

3-2 Algebraic Lagrangian

Let us turn attention to building up the TFT's Lagrangian. We can find the specific relations in the algebra (2.25):

$$\{Q, Q_c\} = -\frac{i}{q} q^z P_z, \quad \{S, S_c\} = \frac{i}{s} s^z K_z, \quad \{Q, S\} = -4i\tilde{M}. \tag{3.1}$$

The above three relations are in the form of $\mathcal{Q}$-exact with the ghost number zero, and the above fermionic operators are nilpotent. Therefore, it may be possible to regard the relations (3.1) as *algebraic* Lagrangians of TFT:

$$\mathcal{L}_Q = \{Q, Q_c\}, \quad \mathcal{L}_S = \{S, S_c\}, \quad \mathcal{L}_{QS} = \{Q, S\}. \tag{3.2}$$

The identifications (3.2) do not seem natural from the relativistic field theoretical view point because $\mathcal{L}_Q$ and $\mathcal{L}_S$ have non-zero spins and scaling dimensions. Intuitively speaking, $\mathcal{L}_Q$ and $\mathcal{L}_S$ are not neutral. We will comment on plausibility and uniqueness of the identificaltions (3.2) in the last two paragraphs of this subsection 3-2.

The three Lagrangians (3.2) are real and must be defined in two dimensional manifolds with boundary. For instance, $\mathcal{L}_{QS}$ is described as two dimensional integration:

$$\mathcal{L}_{QS} = \int_{\partial \mathrm{M}^2} J^0_{\{Q,S\}} = \int_{\mathrm{M}^2} dJ^0_{\{Q,S\}}. \tag{3.3}$$

The existence of boundary is in the way for the following discussions. Notwithstanding, we can construct the quantum theories associated with the Lagrangians (3.2) on manifolds without boundary. It is well known that the path-integral on manifold with boundary describes some states on boundary, which are also topological invariants[1], and consequently we can make the formulation on manifold without boundary by means of the inner product of the path-integrals as "in" and "out" states. Hereafter we then suppose that the manifold of the theory will without boundary [17][18].

Each system of the three Lagrangians (3.2) has the manifest BRST-like fermionic symmetries as follows: The system of $\mathcal{L}_Q$ has Q- and Q_c-symmetries, the system of $\mathcal{L}_S$ has S- and S_c-symmetries and the system of $\mathcal{L}_{QS}$ has Q- and S-symmetries. The energy-stress tensor of each system is a liner-combination of exact-forms of the two fermionic generators which the system has. For instence, the energy-stress tensor of $\mathcal{L}_{QS}$ is

$$T_{\mu\nu} = \{Q, \delta S/\delta g_{\mu\nu}\} + \{S, \delta Q/\delta g_{\mu\nu}\}. \tag{3.4}$$

The equation (3.4) is based on the fact that we have no information whether Q and S are dependent on the metric $g_{\mu\nu}$ or not. It is well known that the correlation functions of $\mathcal{Q}$-exact operators are trivial in TFT[1]:

$$\langle\ \mathcal{Q}\text{-exact}\ \ \rangle = 0, \tag{3.5}$$

where $\langle \cdots \rangle$ means the path-integration. Therefore, the Q- and S-exact form of the energy-stress tensor guarantees that the correlation functions of any non-trivial TFT's observables $\mathcal{O}$ are independent on the metric $g_{\mu\nu}$:

$$\begin{aligned}
\frac{\delta}{\delta g_{\mu\nu}}\langle\ \mathcal{O}\ \rangle &= \langle\ \mathcal{O}T_{\mu\nu}\ \rangle \\
&= \langle\ \mathcal{O}\{Q\text{-exact}\}\ \rangle + \langle\ \mathcal{O}\{S\text{-exact}\}\ \rangle \\
&= \langle\ Q\text{-exact}\ \rangle + \langle\ S\text{-exact}\ \rangle \\
&= 0.
\end{aligned} \tag{3.6}$$

Consequently, we regard Q, S, Q_c and S_c as the topological symmetry genetators in each of the three systems (3.2).

At the classical level, the transformation rules of the three Lagrangians (3.2) under the symmetry generated by the topological algebra (2.25) are as follows:

$$\begin{aligned}
&\delta_{A^\star}\mathcal{L}_Q = 0 \quad \text{up to} \quad Q,\ \ Q_c \text{ -exact terms}, \\
&\delta_{A^\star}\mathcal{L}_S = 0 \quad \text{up to} \quad S,\ \ S_c \text{ -exact terms}, \\
&\delta_{A^\star}\mathcal{L}_{QS} = 0 \quad \text{up to} \quad Q,\ \ S \text{ -exact terms},
\end{aligned} \tag{3.7}$$

where $A^\star$ denotes a set of the generators of the algebra (2.25):

$$A^\star = P_z, K_z, \tilde{M}, \tilde{D}, Q, Q_c, S, S_c. \tag{3.8}$$

The relations (3.7) show that, at the quantum level, every system is invariant under the symmetry which is generated by $A^\star$ because of (3.5).

It is necessary to comment on the above identifications (3.2). In the relativistic field theories, the Lagrangians must be invariant under the local Lorentz transformations. Unfortunately, $\mathcal{L}_Q$ and $\mathcal{L}_S$ in the definitions (3.2) are not invariant under the Lorentz transformations, so that $\mathcal{L}_Q$ and $\mathcal{L}_S$ have integer spin 1. Moreover, they have non-zero scaling dimensions. These facts will lead us to conclude that $\mathcal{L}_Q$ and $\mathcal{L}_S$ are not suitable for Lagrangians from the conventional field theoretical view point. As mentioned above, the path-integrals with such Lagrangians are surely invariant under the local Lorentz and Weyl transformations. Therefore, let us regard $\mathcal{L}_Q$ and $\mathcal{L}_S$ as Lagrangians of TFT.

There is a question whether the other candidates for Lagrangian exist or not. In the algebra (2.25), the remaining relations to be regarded as Lagrangian are all in the same form:

$$[\mathcal{F}_{\text{fermi}},\ \mathcal{B}_{\text{bose}}] = \hat{\mathcal{F}}_{\text{fermi}}. \tag{3.9}$$

While the above commutation relation (3.9) is indeed $\mathcal{Q}$-exact, it is fermiomic. From the physical view point, it is not possible to regard such a fermionic relation as Lagrangian. Clearly, if so, the perturbation could be up to the first order. Moreover, the generators of Poincaré group, which are obtained through Lagrangian with Poincaré group invariace, would be composed of anti-commutators alone and then the crucial relation between the symmetry and the conservation law would be nothing. The Lagrangian formalism (or more generally the mechanics) would break down. Therefore, such a quasi-Lagrangian which is fermionic is by no means accepted as the physical Lagransian. Notwithstanding, there is no reason to disregard the fermionic quasi-Lagrangian if we are allowed formally to regard the quasi-Lagrangian as mere exponent of the Boltzman weight factor in the partition function. What we are interested in is whether, under the zero coupling limit, the classical configuration could be induced or not.

Let us now evaluate the possibility of the fermionic quasi-Lagrangian. In the case of $\mathcal{B}_{\text{bose}} = \tilde{M}$ or $\tilde{D}$, we see $\mathcal{F}_{\text{fermi}} = \hat{\mathcal{F}}_{\text{fermi}}$, that is, the transformation

$$\mathcal{B}_{\text{bose}} : \mathcal{F}_{\text{fermi}} \ \mapsto \ \mathcal{F}_{\text{fermi}} \tag{3.10}$$

is identity endomorphism and then the weight factor of the path-integral becomes the polynomial of $\mathcal{L}$-exact factors:

$$\begin{aligned} \exp(\frac{1}{\alpha}\mathcal{L}) &= 1 + \frac{1}{\alpha}\mathcal{L} \\ &= 1 + \frac{1}{\alpha}[\mathcal{L},\ \mathcal{B}_{\text{bose}}], \end{aligned} \tag{3.11}$$

where α denotes a coupling factor. Therefore, the path-integral with Lagrangian which is the form $[\mathcal{F}_{\text{fermi}},\ \mathcal{B}_{\text{bose}}] = \mathcal{F}_{\text{fermi}}$ yields infinite volume alone and the fact indicates that the path-integral is trivial. The limitation of the coupling factor turns out to be meaningless.

The remaining case is $\mathcal{B}_{\text{bose}} = P$ or K. If we regard the relation (3.9) with $\mathcal{B}_{\text{bose}} = P$ or K as Lagrangians $\mathcal{L}$, then $\mathcal{L}$ is invariant under the transformation generated by $\mathcal{L}$ itself:

$$\delta_{\mathcal{L}}\mathcal{L} = 0. \tag{3.12}$$

In this case, $\mathcal{L}$ is transformed into itself by $\tilde{M}$ and $\tilde{D}$, that is, $\mathcal{L}$ is $\mathcal{L}$-exact:

$$\mathcal{L} = [\mathcal{L},\ \mathcal{B}_{\text{bose}}], \tag{3.13}$$

where $\mathcal{B}_{\text{bose}} = \tilde{M}$ or $\tilde{D}$. The above two facts, that is, the Lagrangian $\mathcal{L}$ is invariant under the transformation generated by $\mathcal{L}$ itself (3.12) and $\mathcal{L}$ is $\mathcal{L}$-exact (3.13), indicate that the path-integral with the Lagrangian $\mathcal{L}$ is also trivial because of the same reason as in the above case of $\mathcal{B}_{\text{bose}} = \tilde{M}$ or $\tilde{D}$. Consequently, the Lagrangian candidates which yield non-trivial path-integrals are $\mathcal{L}_Q$, $\mathcal{L}_S$ and $\mathcal{L}_{QS}$ alone.

3-3 Reduction to Moduli Space

We will show that a moduli space associated with the algebra (2.25) can be derived just by focusing on the Lagrangian $\mathcal{L}_{QS}$ through a gauge system of osp(2|2)$\oplus$osp(2|2) algebra. The same result could be obtained in the case of using the other Lagrangians $\mathcal{L}_Q$ or $\mathcal{L}_S$.

As mentioned above, the configuration of the system results in a corresponding moduli space after the quantization in TFT. The reduction to a moduli space is a result of the weak coupling limit. Under the limit, the leading contribution could be given by zero mode, that is, the classical configuration which makes the Lagrangian vanish. We now suppose that there exists some proper observable which guarantee the non-triviality of the path-integral. Let us start with a vanishing Lagrangian condition. Therefore, we obtain

$$\mathcal{L} = \{Q, S\} = -4i\tilde{M} = 0. \tag{3.14}$$

There exists a system Γ of the gauge fields (2.16) of the topological algebra (2.25). Let a Noether current which generates $\tilde{M}$ be $J_0^{\tilde{M}}$. From the condition (3.14), $dJ_0^{\tilde{M}} = 0$ on 2-manifold M^2 without boundary can be derived. The condition means $J_0^{\tilde{M}}$ is constant on M^2 for arbitrary 2D metric. Moreover, it is necessary to estimate the behavior of the path-integral defined on 2-manifold with boundary under the zero coupling limit. As mentioned above, a path-integral on manifold with boundary can be regarded as a functional on boundary:

$$Z_{\mathrm{D}_i}[\varphi] = \int_{\psi|_{\partial\mathrm{M}^2}=\varphi} \mathcal{D}\psi \, e^{-S(\psi)}, \ \ (i = 1, 2), \tag{3.15}$$

where $\mathrm{M}^2 = \mathrm{D}_1 \cup_{\partial\mathrm{M}^2} \mathrm{D}_2$. The boundary condition $\psi|_{\partial\mathrm{M}^2} = \varphi$ could be confined in a delta function with a proper periodicity. The coupling factor independece of $Z_{\mathrm{D}_i}[\varphi]$ is evident because the invariance of $Z_{\mathrm{D}_i}[\varphi]$ under the gauge transformations by Q and S holds[17]. Therefore, we see that

$$\begin{aligned} Z_{\mathrm{D}_i}[\varphi](\mathcal{O}) &= \int \mathcal{D}\psi \, e^{-S(\psi)} \, \delta_p(\psi|_{\partial\mathrm{M}^2} - \varphi) \, \mathcal{O} \\ &= 0, \end{aligned} \tag{3.16}$$

where $\mathcal{O} = Q$- or S-exact and δ_p denotes a delta function with some proper periodicity. The characterestic (3.16) guarantees the coupling factor independence of $Z_{\mathrm{D}_i}[\varphi]$ in the

similar way to the case of no-boundary. Therefore, the zero coupling limit would induce the classical configuration which makes the Lagrangian vanish and then the condition $J_0^{\tilde{M}} = 0$ on boudary holds because the Lagrangian could be described as 1-dimensional integration on $\partial \mathrm{M}^2$; $\mathcal{L} = \int_{\partial \mathrm{M}^2} J_0^{\tilde{M}}$. Consequently, the condition (3.14) is reduced to $J_0^{\tilde{M}} = 0$ on 2-manifold without boundary. We then define a sub-configuration Γ_s which satisfies $J_0^{\tilde{M}} = 0$. The constraint $J_0^{\tilde{M}} = 0$ yields the following reduction of the configuration:

$$\Gamma \quad \Longrightarrow \quad \Gamma_s. \tag{3.17}$$

Let us confine ourselves to investigation of the physical meaning of the reduction mentioned above just through the Noether current which is composed of the connections of original Osp(2|2)⊗Osp(2|2) ($\equiv \mathcal{G}$). To this aim, we will consider a pure gauge theory of osp(2|2)⊕osp(2|2) , not supergravity theory. Therefore, the naming of the generators of osp(2|2)⊕osp(2|2) , which is shown in Sect.2, is perfectly formal. If not, that is, the naming is meaningful, the general coordinate transformations must be induced, and then the theory becomes empty as in the case of the ordinary 2-dimensional conformal supergravity gauge theory.

Let us start with considering the Yang-Mills action on 2-dimensional manifold without boundary:

$$\mathcal{L}_{\mathrm{YM}_2} = \int_{\mathrm{M}^2} |R_{\mathcal{G}}|^2 * 1, \tag{3.18}$$

where $*$ is Hodge star operator and $R_{\mathcal{G}}$ is a field strength 2-form :$R_{\mathcal{G}} = R^A B I_{AB}$ in which I_{AB} is the Cartan-Killing matrix on the Lie algebra of $\mathcal{G}$. Here the norm $|R_{\mathcal{G}}|^2$ has been obtained by using the metric on M^2 and I_{AB}. It is of interest to argue that the integrand of eq.(3.18) can be rewritten as $R^A \wedge *R^B I_{AB}$, together with the volume form of the metric $*1$. It is a matter of course that eq.(3.18) is invariant with respect to the gauge symmetry $\mathcal{G}$ and has no general coordinate invariance. The time component of the Noether current in association with the symmetry generated by M then turns out to be

$$J_M^0 = \frac{\partial \mathcal{L}_{\mathrm{YM}_2}}{\partial(\partial_0 \mathbf{a}_\mu^A)} G_M^A(\mathbf{a})_\mu = R^B I_{AB} G_M^A(\mathbf{a})_\mu, \tag{3.19}$$

where $G^A_M(\mathbf{a})_\mu$ is defined by

$$\delta_M \mathbf{a}^A_\mu = \lambda_M G^A_M(\mathbf{a})_\mu, \tag{3.20}$$

$\mathbf{a}$ is general form of the gauge connections and $R^A = R^A_{01}$. We can now add the optional field strength components to the original current (3.19) owing to the ambiguity of the Noether current. We are free to choose the additional term:

$$\partial_\alpha(\theta^A R^{A\alpha\beta}), \tag{3.21}$$

where $\theta^A(x)$ 0-form is arbitrary function which supplements the characteristics of J^0_M with respect to the paired field strength R^A. Here note that the summation convention for repeated indices does not apply to indices of θ and of exponent of statistical factor $(-)$ in the following equations. It is a matter of course that the conventional sum rule is alive for the indices except the exponent of θ.

For the purpose of determining the compensating factor $\theta^A(x)$ 0-form, we refer to eq.(3.19) in which $G^A_M(\mathbf{a})_\mu$ is composed of the gauge transformation $\delta_{/\tau}\mathbf{a}^A$ 1-form as in eq.(3.20), and surely correspond to $\theta^A(x)$ 0-form. Therefore, $\theta^A(x)$ 0-form must be constructed through the reduction procedure of $\delta\mathbf{a}^A$ 1-form. That is, we need some map X:

$$X : \delta\mathbf{a}^A\text{...1-form} \quad \mapsto \quad \theta^A\text{...0-form}. \tag{3.22}$$

The map X can indeed be chosen as

$$X \equiv \mathcal{D}^\dagger, \tag{3.23}$$

where $\mathcal{D}^\dagger$ is the adjoint exterior derivative operator:

$$\mathcal{D}^\dagger : \Omega^r(\mathrm{M}^2) \to \Omega^{r-1}(\mathrm{M}^2), \tag{3.24}$$

with $\mathcal{D}^\dagger = *\mathcal{D}*$ on the two dimensional Lorentzian manifold without boundary. We then obtain $\theta^A(x)$ 0-form as follows:

$$\theta^A \equiv \mathcal{D}^\dagger\delta\mathbf{a}^A. \tag{3.25}$$

Accordingly eq.(3.21) is reduced to

$$\partial_\beta(\theta^A R^{A\beta\alpha}) = (\partial_\beta\theta^A - (-)^{|B||A|} f_{BA}{}^C \theta^C \mathbf{a}^B_\beta) R^{A\beta\alpha} + \theta^A \mathcal{D}_\beta R^{A\beta\alpha}, \tag{3.26}$$

where

$$\mathcal{D}_\beta R^{A\beta\alpha} = \partial_\beta R^{A\beta\alpha} + (-)^{|B||C|} f_{BC}{}^A \mathbf{a}^B_\beta R^{C\beta\alpha}. \tag{3.27}$$

We next add eq.(3.26) to J^0_M which leads to

$$J^0_M = \sum_{A(\text{on }\theta)}^{all} [\Pi^{A(\text{on }\theta)}_{\mu=1} R^A + \theta^A \mathcal{D}_{\mu=1} R^A], \tag{3.28}$$

where

$$\Pi^{A(\text{on }\theta)}_{\mu=1} = (-)^{|B||A|} I_{AB} G^B_M(\mathbf{a})_1 + \partial_1\theta^A - \sum_C^{all} (-)^{|B||A|} f_{BA}{}^C \theta^C \mathbf{a}^B_1, \tag{3.29}$$

and $\sum^{all}_{A(\text{on }\theta)}$ denotes a summation of the indices appearing in the exponent of θ. That is, we must sum up the indices A in eq.(3.28) except for the indices appearing in the exponent of the statistical factor $(-)$. The index A of I_{AB} and f^C_{BA} in $\Pi^{A(\text{on }\theta)}_{\mu=1}$ obeys the conventional summation rule.

Our principal task is now to make the topological twist on the current J^0_M (3.28) and set it upon the configuration Γ_s as a result of the weak coupling limit. Making the topological twist on J^0_M leads us to the modified current $J^0_{\tilde{M}}$ where $\tilde{M} = M + 2iV$. The zero field strengths of the current J^0_M are removed through replacement of A by $A^\star$ (3.8), which means that the configuration is reduced to Γ. We then obtain the informations for the limiting condition $J^0_{\tilde{M}} = 0$ as follows:

$$R^{A^\star} = 0, \qquad \theta^{A^\star} = 0. \tag{3.30}$$

Clearly, this solution (3.30) is not unique in the mathematical view point, but seems natural because of the independence of the specific space-time coordinate index: $\mu = 1$.

The informations $R^{A^\star} = 0$ and $\theta^{A^\star} = 0$ obtained above play the roles of the constraints for the configuration Γ_s, which eventually lead to some moduli space. Number of the equivariant constraint $R^{A^\star} = 0$ is equal to that of the fermionic connections with ghost number $\psi_\mu(-1)$, $\psi_{\mu-}{}^+(1)$, $\phi_\mu(1)$ and $\phi_{\mu-}{}^+(-1)$. Therefore, $R^{A^\star} = 0$ can be regarded as the fixing condition of the so-called topological symmetry whose degrees of

freedom is equal to number of these fermionic connections, i.e. the so-called topological ghosts.

Let us next explain the physical meaning of the condition $\theta^{A^\star} = 0$. The tangent of the connection space $\mathcal{A}$ can be decomposed[19] as follows:

$$T_{\mathbf{a}}\mathcal{A} = Im\mathcal{D} \oplus Ker\mathcal{D}^\dagger, \tag{3.31}$$

where $Im\mathcal{D}$ 1-form is the tangent in the gauge direction, while $Ker\mathcal{D}^\dagger$ 1-form means the component orthogonal to the gauge orbit and $0 = \theta^{A^\star} = \mathcal{D}^\dagger\delta\mathbf{a}^{A^\star}$ is natural gauge condition in which $\delta\mathbf{a}^{A^\star}$ is infinitesimal variation of the connection $\mathbf{a}^{A^\star}$. The constraint $\theta^{A^\star}$, the number of which is equal to that of the generators of the gauge symmetry, is then regarded as the gauge fixing condition.

We can therefore claim that all the constraints $R^{A^\star} = 0$, $\theta^{A^\star} = 0$ which originate from $J^0_{\tilde{M}} = 0$ indeed lead to a moduli space of flat connections:

$$\mathcal{M}_{flat} = \{R^{A^\star} = 0\}/\mathcal{G}^\star. \tag{3.32}$$

The moduli space (3.32) is really associated with the topological algebra (2.25). The BRST gauge fixing is necessary, by way of parenthesis, for the detailed investigation of observables, correlation functions and their geometrical meaning in TFT[20][21]. Incidentally let us describe another representation for the conditions (3.30). If the infinitesimal variation of the connection $\delta\mathbf{a}^{A^\star}$ are on Γ_s, the variations of $R^{A^\star}$ under $\delta\mathbf{a}^{A^\star}$ must also vanish. Linearized representation[22][23] of the flat connection equations yields

$$\begin{aligned} &0 = *\delta R = *\mathcal{D}\delta\mathbf{a} = *\mathcal{D} * *\delta\mathbf{a} = \mathcal{D}^\dagger * \delta\mathbf{a}, \\ &0 = \mathcal{D}^\dagger\delta\mathbf{a}. \end{aligned} \tag{3.33}$$

If $\delta\mathbf{a}$ is the arbitrary variation on $\mathcal{M}_{flat}$, its Hodge dual $*\delta\mathbf{a}$, which is still 1-form only in the two-dimension, is also on $\mathcal{M}_{flat}$.

Let us next refer to the general coordinate transformations. In the ordinary N=2 conformal supergravity, all curvatures must vanish in full consonance with the general coordinate transformations as gauge symmetry generated by the conformal super

group[10]. As a consequence, there exists no kinetic term, i.e. no dynamics of connection fields in the ordinary theory. In the present case, on the contrary, zero curvatures play the roles of the conditions which lead to the configuration of the fields. Accordingly, the general coordinate transformation $\delta_{gc}(\xi)$ is induced by these conditions. $\delta_{gc}(\xi)$ expressed as

$$\delta_{gc}(\xi)\mathbf{a}_\mu^A = \sum_B \delta_B(\xi^\nu \mathbf{a}_\nu^B)\mathbf{a}_\mu^A + \xi^\nu R_{\nu\mu}^A. \tag{3.34}$$

The topological twist on eq.(3.34) induces the replacement $A \rightarrow A^\star$, so that the zero field strengths associated with $A - A^\star$ are removed. Moreover, under the weak coupling limit, the resultant configuration is given by (3.30). Therefore, the transformation law (3.34) on $\mathcal{M}_{flat}$ is described as follows:

$$\delta_{gc}(\xi)\mathbf{a}_\mu^{A^\star} = \sum_{B^\star} \delta_{B^\star}(\xi^\nu \mathbf{a}_\nu^{B^\star})\mathbf{a}_\mu^{A^\star}, \tag{3.35}$$

In eqs.(3.35), $\delta_{gc}(\xi)$ have been fixed in accordance with the thoroughly fixed gauge symmetry (3.32). It is then possible to argue that the configuration $\mathcal{M}_{flat}$ is a quotient not only in the sense of the gauge symmetry, but also in the sense of the diffeomorphism:

$$\sim /\mathcal{G}^\star \quad \supset \quad \sim /\mathrm{Diff}_0. \tag{3.36}$$

4 Geometrical Meaning of Fermionic Operators

In the last section, the moduli space $\mathcal{M}_{flat}$ (3.32) has been derived formally. It is possible to obtain more informations on $\mathcal{M}_{flat}$ by studying a geometrical meaning of the fermionic operators of the algebra (2.25). In TFT, the operator δ_f of the BRST-like fermionic symmetry corresponds to the exterior derivative operator d on a moduli space where the ghost number corresponds to the form degree. In the topological Yang-Mills theory on 4-manifolds[1][8], for instance, the cotangent vector, i.e. 1-form on the

Yang-Mills instanton moduli space, is described as

$$\delta_f \mathbf{a} = \psi, \tag{4.1}$$

where $\mathbf{a}$ is a generic point in the moduli space and ψ is a topological ghost. In the present case, it is natural to regard the fermionic operators Q, S, Q_c S_c as δ_f, because there exists the ghost number which has nothing to do with the gauge symmetry and moreover the fermionic operators generate the transformations with the ghost number, that is, they are the ghost number-carriers. Whatever the gauge orbit may be collapsed under the zero coupling limit, the four fermionic operators can still remain as BRST-like operators on $\mathcal{M}_{flat}$. The operation on $\mathcal{M}_{flat}$ must be read off from the transformation rule (2.19). The existence of such operators on $\mathcal{M}_{flat}$ leads us to consistent and interesting results. A space which we can regard as a moduli space will be equiped with some analytical, or in other word, differentiable structure, in general. It is well-known, for instance, that the moduli space of 4D (anti-) instantons is locally homeomorphic to a differentiable manifold under the appropriate conditions and the complex structure of the (anti-) instanton moduli space corresponds to that of the basemanifold [24][12]. It is a matter of course that the topological invariants of TFT, which are originated from the Donaldoson theory, must be integrals on certain analytical support of the moduli space.

We now suppose that the moduli space $\mathcal{M}_{flat}$ (3.32) has such an analytical support. This assumption is appropriate becouse the general discussion of flat connections shows that the moduli space of flat connections is regarded as a manifold. In the present case, it is possible to decompose $\mathcal{M}_{flat}$ locally into fermionic sub-space $\mathcal{M}^f$ and bosonic sub-space $\mathcal{M}^b$ as follows:

$$\mathcal{M}_{flat} \cong \mathcal{M}^f \otimes \mathcal{M}^b. \tag{4.2}$$

Accordingly, $\mathcal{M}_{flat}$ is regarded as a fiber bundle over the base space $\mathcal{M}^b$, which is described by using the following fibration:

$$\begin{array}{ccc} F & \longrightarrow & \mathcal{M}_{flat} \\ & & \big\downarrow \pi \\ & & \mathcal{M}^b \end{array} \tag{4.3}$$

F denotes a fiber parametrized by the ghost number. That is, $\mathcal{M}_{flat}$ is regarded as the Whitney sum bundle composed of the vector bundles; E_k with the ghost number $k(=-1,0,1)$:

$$\mathcal{M}_{flat} = E_{-1} \oplus E_0 \oplus E_1. \tag{4.4}$$

Let us discuss a geometrical meaning of the fermionic operators in the present theory. In sect.3, the three TFT's Lagrangians have been set up in (3.2). While, under the weak coupling limit, we have induced the reduction to the moduli space $\mathcal{M}_{flat}$ by considering one of the three Lagrangians; $\mathcal{L}_{QS}$, all Lagrangians are adapted here. We introduce a total Lagrangian $\mathcal{L}_{tot}$ as a linear combination of the three Lagrangians:

$$\mathcal{L}_{tot} = \mathcal{L}_Q + \mathcal{L}_S + \mathcal{L}_{QS}. \tag{4.5}$$

Using the algebra (2.25), $\mathcal{L}_{tot}$ is also described as an anti-commutator of two fermionic operators as follows:

$$\mathcal{L}_{tot} = \{\mathcal{QS}, \mathcal{QS}^\dagger\}, \tag{4.6}$$

where $\mathcal{QS} = Q + S_c$ and $\mathcal{QS}^\dagger = Q_c + S$. As can be seen from the relations (2.21), $\mathcal{QS}$ increases the ghost number by one unit, while $\mathcal{QS}^\dagger$ decreases it by the same quantity. Moreover, both $\mathcal{QS}$ and $\mathcal{QS}^\dagger$ are nilpotent. Under the weak coupling limit, $\mathcal{QS}$ and $\mathcal{QS}^\dagger$ can be regarded as operators on the fiber bundle $\mathcal{M}_{flat}$. To be precise, $\mathcal{QS}(\mathcal{QS}^\dagger)$ operates on the vector bundles $E_k(k=-1,0,1)$. The operation sequence of $\mathcal{QS}$ is

$$0 \quad \stackrel{i}{\longrightarrow} \quad E_{-1} \quad \stackrel{\mathcal{QS}_{-1}}{\longrightarrow} \quad E_0 \quad \stackrel{\mathcal{QS}_0}{\longrightarrow} \quad E_1 \quad \stackrel{\mathcal{QS}_1}{\longrightarrow} \quad 0, \tag{4.7}$$

where $E_{-1} = \{\psi, \phi^z\}$, $E_0 = \{\omega, b, e^z, f^z\}$, $E_1 = \{\psi^z, \phi\}$, $\mathcal{QS}_k$ $(k=-1,0,1) = \mathcal{QS}$ and i denotes inclusion. Therefore, we can regard the above sequence (4.7) as an elliptic complex, and $\mathcal{QS}$ as a Fredholm operator with $\mathcal{QS}^\dagger$ its adjoint. Accordingly, $\mathcal{L}_{tot}$ corresponds to the Laplacian operator.

It is possible to derive the index of the above elliptic complex as an additional result:

$$\begin{aligned} \mathrm{ind}(\mathcal{QS}) &= \sum_{k=-1}^{1} (-1)^k \mathrm{Harm}^k(\mathcal{M}_{flat}, \mathcal{QS}) \\ &= 0, \end{aligned} \tag{4.8}$$

where $\mathrm{Harm}^k(\mathcal{M}_{flat},\ \mathcal{QS})$ denotes the Kernel of $\{\mathcal{QS}_k, \mathcal{QS}_k^\dagger\}$. The above discussions may be associated with the Morse theory applied to the supersymmetric NL-σ model[25] by E. Witten. In Ref.[25], the Hamiltonian is composed of the supercharges which correspond to the coboundary operator and its adjoint of the de Rham theory and play a crucial role for studying the supersymmetry breaking. Clearly, $\mathcal{L}_{tot}$ does not correspond to the Hamiltonian H in contrast with the theory in Ref.[25], because the ghost number runs from -1 to $+1$ and then $\mathcal{QS}$ can not be identified with the exterior derivative operator of the de Rham theory. Therefore, the index and the harmonic forms in (4.8) can not be also identified with the Euler number and the Betti numbers, respectively.

The vanishing Lagrangian condition $\mathcal{L}_{tot} = 0$ means that the eigenvalues of the Laplacian operator must be zero. The fact seems to show that $\mathcal{M}_{flat}$ is corresponding to the Kernels; $\mathrm{Harm}^k(\mathcal{M}_{flat}, \mathcal{QS})$. To see the contents of the Kernels, we describe the operator sequence (4.7) in terms of the components:

$$
\begin{array}{ccccc}
\phi^z & \overset{Q}{\underset{S}{\rightleftarrows}} & f^z & \xrightarrow{S_c} & \phi \\[2ex]
\psi & \xleftarrow{S} & \omega & \xrightarrow{Q} & \phi \\[2ex]
\psi & \xleftarrow{Q_c} & e^z & \overset{Q}{\underset{S}{\rightleftarrows}} & \psi^z \\[2ex]
 & & b & &
\end{array}
\tag{4.9}
$$

As can be seen from the sequences (4.9), four components e^z, f^z, ψ^z and ϕ^z are not suitable for the Kernel of the Laplacian. In the above context, we have indicated that $\mathcal{L}_{tot}$ is regarded as the Laplacian under the weak coupling limit and the vanishing Lagrangian condition leads us to the Kernels of the Laplacian. It is natural to expect that the Kernels correspond to $\mathcal{M}_{flat}$. Consequently, we claim that the four components must not be in $\mathcal{M}_{flat}$.

This statement is also supported by the following consideration. First of all, there

are two closed loops in the sequences (4.9), which are composed of two sets (e^z, ψ^z), (f^z, ϕ^z):

$$\phi^z \underset{S}{\overset{Q}{\rightleftharpoons}} f^z \quad , \quad e^z \underset{S}{\overset{Q}{\rightleftharpoons}} \psi^z. \tag{4.10}$$

The two components in each set circulate on its own loop by operation of Q and S. Any other componets can not reach the two loops by any operations. Moreover, the S-operation after the Q-operation (or Q after S) leads the component to itself and such a double operation of $S * Q$ (or $Q * S$) induces the transformation in the direction of the gauge orbit, because of the relation $[Q, S] = -4i\tilde{M}$. Therefore, the two closed loops are in the gauge orbit and are expected to be collapsed together with the gauge orbit on $\mathcal{M}_{flat}$. The fact shows that e^z, f^z, ψ^z and ϕ^z are not in the muduli space $\mathcal{M}_{flat}$.

More detailed informations of $\mathcal{M}_{flat}$ are obtained, through studying the geometrical meaning of the fermionic operators. It is clarified that the choice of the left-chiral part of the algebra (2.25) is rather meaningless for the purpose of the derivation of the moduli space and the intersection of the left- and right-chiral part is only effective. Therefore, $\mathcal{M}_{flat}$ is reduced to $\mathcal{M}_0$:

$$\mathcal{M}_0 = \{R^{A^{\star\star}} = 0\}/\mathcal{G}^{A^{\star\star}}, \tag{4.11}$$

where $A^{\star\star} = \tilde{M}, \tilde{D}, Q, S$. We then claim that the moduli space intrinsic to the topologically twisted osp(2|2)$\oplus$osp(2|2) is really $\mathcal{M}_0$.

4-1 Observable

Let us mention the triviality of the path-integral. The triviality depends on the existence and the characterestics of observables. From the view point of the ordinary quantum gauge theory, the condition for observable is the gauge invariance. In TFT, another condition is required, i.e. metric independence as well as gauge invariance. The above two conditions would not be enough to yield non-trivial correlation functions. The fermionic contribution in the path-integral measure must not be disregarded. If fields are assigned by ghost number, the path-integral measure may have ghost number

anomaly. Therefore, to absorb the ghost number of the measure, observables must have the same ghost number as the measure.

In the present case, while we see that the measure is free from the ghost number anomaly, the measure on the moduli space $\mathcal{M}_0$ includes the fermionic contributions $d\psi\, d\phi$ and the difficulty of Grassman number integration is retained. If the integrand does not include the coupling $\psi\, \phi$, the integration on $\mathcal{M}_0$ is zero. After all, the triviality of the path-integral depends on the existence of observables which are gauge invariant, metric independent and including the coupling $\psi\phi$, rigorously speaking, gauge invariance and metric independence modulo $\mathcal{Q}$-exact.

Now let us find the observable which satisfies the above three conditions. We note that the TFT's Lagrangians (3.2) are invariant under the gauge symmetry generated by the algebra (2.25) composed of $A^\star$ (3.8) at the quantum level. First of all, we must pay attention to the Yang-Mills action associated with the algebra $A^\star$:

$$\mathcal{O}_{\mathrm{YM}_2} = \int_{\mathrm{M}^2} I_{AB} R^A \wedge *R^B, \tag{4.12}$$

where $A(B)$ is the label of the algebra $A^\star$. The above I_{AB} denotes the Cartan-Killing matrix:

$$I_{AB} = -(-)^{|A|+|C|(|A|+|B|+1)} f_{AD}{}^C f_{BC}{}^D. \tag{4.13}$$

The Yang-Mills action in two dimensions:

$$\mathcal{I}_{\mathrm{YM}_2} = \int_{\mathrm{M}^2} d\mu g^{ai} g^{bj} \mathrm{Tr}(F_{ab} F_{ij}) \tag{4.14}$$

do not depend on a metric g in general[26]. The metric dependence is up to the measure $d\mu$ on the two dimensions M^2. A curvature two form R can be written as $R = \varepsilon_v \hat{R}$ where ε_v denotes an two form determined by a metric and $\hat{R}$ is an algebra-valued zero form. Using the zero form $\hat{R}$, the Yang-Mills action $\mathcal{I}_{\mathrm{YM}_2}$ can be described as

$$\mathcal{I}_{\mathrm{YM}_2} = \int_{\mathrm{M}^2} d\mu I_{AB} \hat{R}^A \hat{R}^B. \tag{4.15}$$

Therefore, we can regard $\mathcal{I}_{\mathrm{YM}_2}$ as metric independent if $d\mu$ and $\hat{R}$ do not contain a metric. It is a matter of course that the requirement of coordinate transformation invariance of $d\mu$ would induce metric dependence of $d\mu$, however.

Now our concentration on the Yang-Mills action $\mathcal{O}_{\mathrm{YM}_2}$ (4.12) will be recoverd. We then evaluate the Cartan-Killing matrix I_{AB}. Number of the generators of $A^\star$ is 8 and number of the independent elements of the Cartan-Killing matrix I_{AB} is then 28. Using the commutation relations (2.25) of the algebra $A^\star$ and the definition (4.13) of I_{AB}, we can see that the Cartan-Killing matrix I_{AB} is zero matrix. Therefore, the Yang-Mills action becomes zero and we must find another candidate for observable.

Let us consider next candidate. The metric independent integral (4.15) will be still useful. If $I_{AB} \equiv 1$, the two dimensional integration

$$\mathcal{O}_{\hat{R}^A\hat{R}^B} = \int_{\mathrm{M}^2} d\mu \hat{R}^A \hat{R}^B \tag{4.16}$$

will not be invariant under the gauge symmetry, in contrast to the Yang-Mills action. Notwithstanding, we expect that some $\mathcal{O}_{\hat{R}^A\hat{R}^B}$ will be gauge invariant modulo $\delta_{\mathcal{Q}}$-exact. First of all, we must pay attention to the fact that all $\hat{R}^A$ can be represented as $\mathcal{Q}$-exact form:

$$\hat{R}^A \sim \delta_{\mathcal{Q}} \hat{R}^B, \tag{4.17}$$

where the RHS of eq.(4.17) is not necessarily unique due to the algebra $A^\star$ (2.25). For instance, $\hat{R}^{\tilde{M}} \sim \delta_{Q_c} \hat{R}^{Q_c} \sim \delta_{S_c} \hat{R}^{S_c}$. The $\mathcal{Q}$-exact form (4.17) leads to the fact that all quadratic forms $\hat{R}^A\hat{R}^B$ except for $\hat{R}^P\hat{R}^K$ and $\hat{R}^{Q_c}\hat{R}^{S_c}$ can be also described as the $\mathcal{Q}$-exact form:

$$\hat{R}^A\hat{R}^B \sim \delta_{\mathcal{Q}} \hat{R}^C\hat{R}^D. \tag{4.18}$$

The two quadratic forms $\hat{R}^P\hat{R}^K$ and $\hat{R}^{Q_c}\hat{R}^{S_c}$ are related as follows:

$$\hat{R}^P\hat{R}^K \sim \delta_{\mathcal{Q}_1}(\hat{R}^{Q_c}\delta_{\mathcal{Q}_2}\hat{R}^{S_c}) + a\hat{R}^{Q_c}\hat{R}^{S_c}, \tag{4.19}$$

where a is some constant determined by the algebra $A^\star$. $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are i) Q and S, respectively, or ii) S and Q, respectively. Then we see that $\mathcal{O}_{\hat{R}^P\hat{R}^K} \sim \mathcal{O}_{Q_cS_c}$ in the path-integral. While $\mathcal{O}_{\hat{R}^P\hat{R}^K}$ (or equivalently $\mathcal{O}_{\hat{R}^{Q_c}\hat{R}^{S_c}}$) is non-trivial and metric independent, unfortunately $\mathcal{O}_{\hat{R}^P\hat{R}^K}$ is not gauge invariant (mudulo $\delta_{\mathcal{Q}}$-exact):

$$\delta_g \hat{R}^P\hat{R}^K \sim b\hat{R}^P\hat{R}^K + \delta_{\mathcal{Q}}\text{-exact}, \tag{4.20}$$

where b is some constant determined by the algebra $A^\star$. Therefore, the second candidate of the form $\mathcal{O}_{\hat{R}^A\hat{R}^B}$ can not be accepted as observable.

Our final trial to obtain the non-trivial observable will be as follows. The first candidate $\mathcal{O}_{\mathrm{YM}_2}$ occur easily to us, which results to be zero; $\mathcal{O}_{\mathrm{YM}_2} = 0$. The second candidate is the modification of the first candidate. On the contrary, the final candidate which we will show is somewhat heuristic. Let us begin with introducing the following two forms:

$$\varrho_\alpha = \varepsilon_v \hat{\varrho}_\alpha, \ \ (\alpha = 0, 1). \tag{4.21}$$

ε_v denotes the above mentioned two form determined by a metric and $\hat{\varrho}_\alpha$ is zero form defined by

$$\hat{\varrho}_\alpha = \psi_\alpha \phi_\alpha. \tag{4.22}$$

The index α can be regarded as mere labels because the coordinate transformations are not considered. Therefore, we present the final candidate as in the following:

$$\mathcal{O}_\varrho = \int_{\mathrm{M}^2} d\mu \ \varrho_0 \varrho_1. \tag{4.23}$$

The behavior of ϱ_α under the gauge transformation δ_g (2.19) is written in the form:

$$\begin{aligned} \delta_g \varrho_\alpha &= (\partial_\alpha \epsilon)\phi_\alpha + \psi_\alpha \partial_\alpha \kappa \\ &= \acute{\epsilon}\phi_\alpha - \acute{\kappa}\psi_\alpha \\ &= \delta_{QS}\ \omega_\alpha, \end{aligned} \tag{4.24}$$

where $\acute{\epsilon} = \partial_\alpha \epsilon$ and $\acute{\kappa} = \partial_\alpha \kappa$. Therefore, we see that

$$\begin{aligned} \delta_g \mathcal{O}_\varrho &= \int_{\mathrm{M}^2} d\mu \ \delta_g(\varrho_0 \varrho_1) \\ &= \int_{\mathrm{M}^2} d\mu \{(\delta_g \ \varrho_0)\varrho_1 + \varrho_0(\delta_g \ \varrho_1)\} \\ &= \int_{\mathrm{M}^2} d\mu \{(\delta_{QS}\ \omega_0)\varrho_1 + \varrho_0(\delta_{QS}\ \omega_1)\} \\ &= \int_{\mathrm{M}^2} d\mu \ \delta_{QS}\ (\omega_0\varrho_1 + \varrho_0\omega_1 - \omega_0\omega_1) \\ &= \delta_{QS} \int_{\mathrm{M}^2} d\mu(\omega_0\varrho_1 + \varrho_0\omega_1 - \omega_0\omega_1). \end{aligned} \tag{4.25}$$

The above eq.(4.25) shows that $\mathcal{O}_\varrho$ is gauge invariant in the path-integral. Clearly, $\mathcal{O}_\varrho$ is not dependent on a metric and includes the coupling $\psi_0\,\phi_0\,\psi_1\,\phi_1$. Therefore, we conclude that $\mathcal{O}_\varrho$ is a non-trivial TFT's observable. The fact shows that the path-integral is not trivial and supports the present discussion from beginning to end.

5 Summary and Remarks

First: We have investigated the topologically twisted osp(2|2)⊕osp(2|2) conformal superalgebra and derived the moduli space intrinsic to the twisted algebra. The algebra includes the appropriate TFT's Lagrangians composed of the fermionic charges Q, S, Q_c and S_c. They lead us to the moduli space $\mathcal{M}_{flat}$ intrinsic to the algebra under the condition of the weak coupling limit. As consequence of the investigation of the geometrical meaning of the fermionic charges, it is shown that $\mathcal{M}_{flat}$ is reduced to $\mathcal{M}_0$ associated with the intersection of the left- and right-chiral part of the topological algebra and is just a moduli space inherent to the algebra. As an additional result, the index of these fremionic operators is derived if some proper support in the moduli space can be defined. The facts which have been clarifed in the above discussion show that the topological algebra has a specific relation with a moduli problem. It is claimed that a geometrical feature of the algebra is one of the interesting characteristics inherent to the topological twist. Therefore, we has succeeded in shedding some light upon the relation between the topological twist and the moduli problem through the geometrical aspect of the topological algebra. *Secondly*: Let us make a remark on the vanishing Noether current. Making use of the ambiguity of the Noether current, we are led to the relation (3.28) between the vanishing Noether current $J^0_{\tilde{M}} = 0$ and the flat connection conditions $R^{A^\star} = 0, \theta^{A^\star} = 0$. In the conventional QFT, e.g. the quantum electro dynamics, this ambiguity mentioned above plays an important role in association with avoiding one mass-less state to obtain a well-defined conserved charge while its physical role is not clarified in the case of the classical correspondent. In the present TFT, on the contrary,

the classical theory has been obtained as the limiting case of the path-integral, and consequently the ambiguity argued above leads to the corresponding moduli problem.

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References

[1] E. Witten, Commun. Math. Phys. **117**, 353(1988).

[2] See, for example, D. Birmingham, M. Blau, M. Rakowski and G. Thompson, Phys. Rep. **209**, 129(1991), and references therein.
For recent developments, see, S. Cordes, G. Moore, and S. Ramgoolam, *Lectures on 2D Yang-Mills Theory, Equivariant Cohomology and Topological Field Theories*, YCTP-p11-94, hep-th/9411210, and references therein.

[3] T. Eguchi and S.-K. Yang, Mod. Phys. Lett. **A5**, 1653(1990).
T. Eguchi, S. Hosono and S.-K. Yang, Commun. Math. Phys. **140**, 159(1991).

[4] P. Candelas, M. Lynker and R. Schimmrigk, Nucl. Phys. **B341**, 383(1990).
M. Lynker and R. Schimmrigk, Phys. Lett. **B249**, 237(1990).
B. R. Greene and R. Plesser, Nucl. Phys. **B338**, 15(1990).

[5] H-L. Hu, Phys. Rev. **D46**, 1761(1992).

[6] E. Witten, Nucl.Phys. **B371**, 191(1992).

[7] Y. Kazama and H. Suzuki, Nucl. Phys. **B321**, 232(1989).

[8] E. Witten, Int. J. Mod. Phys. **A6**, 2775(1991).

[9] J. Sonnenschein, Phys. Rev. **D42**, 2080(1990).

[10] P. van Nieuwenhoizen, Int. J. Mod. Phys. **A1**, 155(1986).
M.Hayashi, S. Nojiri and S. Uehara, Z. Phys. **C31**, 561(1986).

[11] J. M. F. Labastida and P. M. Llatas, Nucl. Phys. **B379**, 220(1992).

[12] A. Galperin and O. Ogievetsky, Commun. Math. Phys. **139**, 377(1991).

[13] V. G. Kac, Commun. Math. Phys. **53**, 31(1977).
P. G. O. Freund and I. Kalansky, J. Math. Phys. **17**, 228(1976).
W. Nahm and M. Scheunnert, J. Math. Phys. **17**, 868(1976).

[14] J. MacCabe and B. Velikson, Phys. Rev. **D40**, 400(1989).

[15] E. Witten, *Mirror Manifolds and Topological Field Theory*, in: Essays on Mirror Manifolds (International Press, 1992), hep-th/9112056.

[16] P. Aspinwall and D. Morrison, Commun. Math. Phys. **151**, 245(1993).

[17] See, for example, G. Thompson, *1992 Trieste Lectures on Topological Gauge Theory and Yang-Mills Theory*, in: Proc.High Energy Physics and Cosmology (Trieste, 1992), hep-th/9305120, and references therein.

[18] E. Witten, Nucl. Phys. **340**, 281(1990)

[19] O. Babelon and C. M. Viallet, Commun. Math. Phys. **81**, 515(1981).

[20] H. Kanno, Z. Phys. **C43**, 477(1989).

[21] L. Baulieu and I. Singer, Nucl. Phys. (Proc. Suppl.)**5B**, 12(1988).

[22] M. Atiyha and R. Bott, Philos. Trans. R. Soc. London **A308**, 523(1982).

[23] N. J. Hitchin, Proc. London Math. Soc. **55**, 59(1987).

[24] M. Itoh, J. Math. Soc. Phys. Jpn. **40**, 9(1988).

[25] E. Witten, Nucl. Phys. **B202**, 253(1982) ; J. Diff. Geom. **17**, 661(1982).

[26] See, for example, E. Witten, Commun. Math. Phys. **141**, 153(1991).